Sensor Aided Inertial Navigation Systems

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Outline

Today, I will discuss:

- **Inertial Navigation System.**
- Carrier phase differential GPS (CDGPS) - INS.
- CDGPS - Vision - INS.
- Stationary updates - INS.
- Near Real Time estimation.

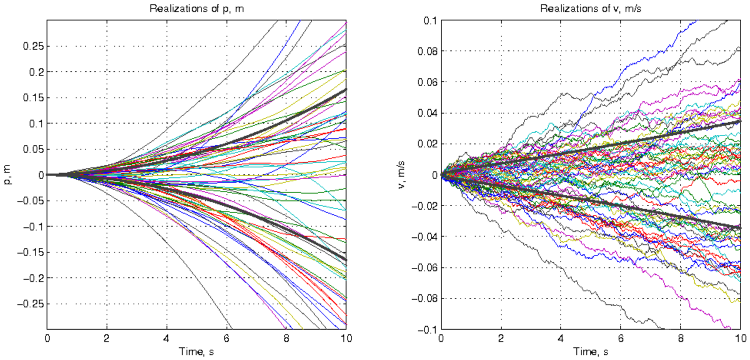
Inertial Sensors

Popularity of inertial Sensors are due to:

- Immunity to jamming, No external reference required (in theory).
- Advent of MEMS sensors [1]:
  - Low cost, Small footprint ($\sim 22 \times 33 \times 11$ mm).
  - Well understood and quantifiable error models [5], [8], [12].
  - High frequency updates ($>$ 100 Hz), High Bandwidth ($>$ 330 Hz), High operating ranges ($\sim \pm$400 deg/s, $\pm 10g$ m/s/s).
- Supplies full 6 degrees-of-freedom pose information.
- Consumer driven demand for applications such as
  - Navigation: routing, vehicle guidance & control [4], [19] etc.
  - High accuracy mobile mapping [18], [17].
  - Life-critical systems: Vehicle collision avoidances, automotive air bags etc.
  - Hand-held devices: Cellphones, Cameras, Electronics readers etc.

Inertial Sensors

- Potentially unbounded error growth in dead reckoning Inertial systems.



Aiding Sensors

"GPS, Vision, LIDAR, Magnetometer, Stationary updates etc.,".

- Features usually complement inertial sensors:
  - Independent and bounded long term errors.
  - Low update frequency.
  - Do not provide 6-DOF information.
- Aided INS problems can be formulated and solved under the Bayesian framework:
  - Extended Kalman Filters, Particle Filters, Unscented Kalman Filters etc.,

Inertial Navigation System

- Navigation state:

$$\boldsymbol{x}^\top = \begin{bmatrix} {}^n\boldsymbol{p}_{nb}^\top & {}^n\boldsymbol{v}_{nb}^\top & {}^n_b\mathfrak{q}^\top \end{bmatrix}, \ \boldsymbol{x} \in \mathbb{R}^6 \times \mathbb{S}^3.$$

- Inertial Measurements: ${}^b\tilde{\boldsymbol{u}} = {}^b\boldsymbol{u} + {}^b\boldsymbol{b} + \boldsymbol{n}, \ \ {}^b\boldsymbol{u} \in \mathbb{R}^6.$
- Bias Gauss-Markov model: ${}^b\dot{\boldsymbol{b}} = -\Lambda {}^b\boldsymbol{b} + \boldsymbol{n}_b.$
- Kinematic equations: $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$

$$\begin{aligned} {}^n\dot{\boldsymbol{p}}_{nb} &= {}^n\boldsymbol{v}_{nb} \\ {}^n\dot{\boldsymbol{v}}_{nb} &= {}^n_b\boldsymbol{R}\,{}^b\boldsymbol{f}_{ib} - {}^n\boldsymbol{g} - 2\left[{}^n\omega_{ie}\times\right]{}^n\boldsymbol{v}_{nb} \\ {}^n_b\dot{\boldsymbol{R}} &= {}^n_b\boldsymbol{R}\left(\left[{}^b\omega_{ib}\times\right] - \left[{}^b\omega_{ie}\times\right]\right) \end{aligned}$$

- INS augmented error state: $\delta\bar{\mathbf{x}}^\top = \begin{bmatrix} \delta\boldsymbol{x}^\top & {}^b\delta\boldsymbol{b}_g^\top & {}^b\delta\boldsymbol{b}_a^\top \end{bmatrix} \in \mathbb{R}^{15}.$
- Linearized error propagation model: $\delta\dot{\bar{\mathbf{x}}} = \boldsymbol{A}\delta\bar{\mathbf{x}} + \boldsymbol{G}\boldsymbol{N}.$

Frames: n=navigation, e=ECEF, i=inertial, b=body, c=camera

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Carrier Phase Differential GPS - INS

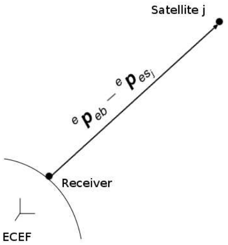
Tightly coupled GPS-INS is the *de-facto* standard for outdoor navigation:

- Performance with double differenced, differential carrier-phase processing with differential corrections: [6, 7, 8],.
  - 1σ positioning accuracy in the order of $0.01 - 0.1$ m.
  - 1σ attitude accuracy in the order of 1 deg.
- Well understood conditions to achieve full state observability [3, 8, 9, 10, 11, 16].
- Updates with a minimum of 2 satellite measurements (Loosely coupled needs at least 4).
- Can do sequential updates ($\boldsymbol{HPH}^{\top} + \boldsymbol{R}$ is a scalar).

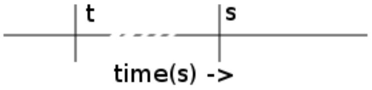
Carrier Phase Differential GPS - INS

Generic GPS measurement model:

$$
\begin{aligned}
\tilde{\rho}^j_k &= ||{}^e\mathbf{p}_{eb} - {}^e\mathbf{p}_{es_j}|| + \nu^j_\rho &;\quad \nu^j_\rho \sim \mathcal{N}(0, 0.5^2)\\
\tilde{\phi}^j &= ||{}^e\mathbf{p}_{eb} - {}^e\mathbf{p}_{es_j}|| + \lambda N^j + \nu^j_\phi &;\quad \nu^j_\phi \sim \mathcal{N}(0, 0.01^2)\\
\tilde{D}^j &= \frac{d}{dt}||{}^e\mathbf{p}_{eb} - {}^e\mathbf{p}_{es_j}|| + \nu^j_D &;\quad \nu^j_\phi \sim \mathcal{N}(0, 0.02^2)
\end{aligned}
$$

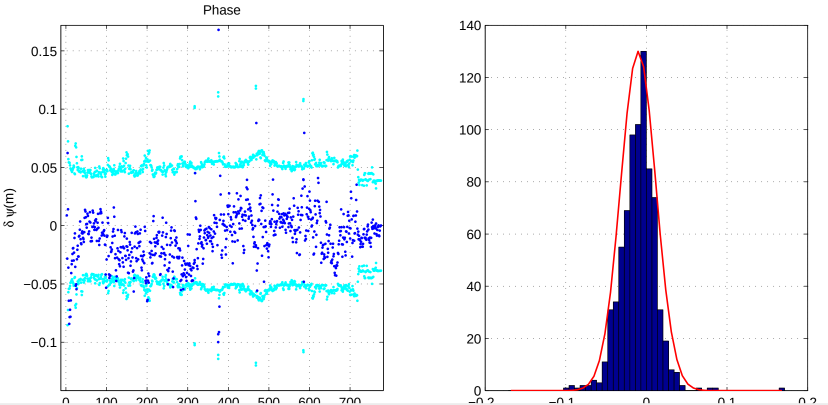


$$
\begin{aligned}
\delta\phi^j &= \tilde{\phi}^j - \hat{\phi}^j_k = ||{}^e\mathbf{p}_{eb} - {}^e\mathbf{p}_{es_j}|| - ||{}^e\hat{\mathbf{p}}_{eb} - {}^e\mathbf{p}_{es_j}|| + \nu^j_\phi\\
&= \mathbf{h}^j\delta\mathbf{x} + \nu^j_\phi
\end{aligned}
$$



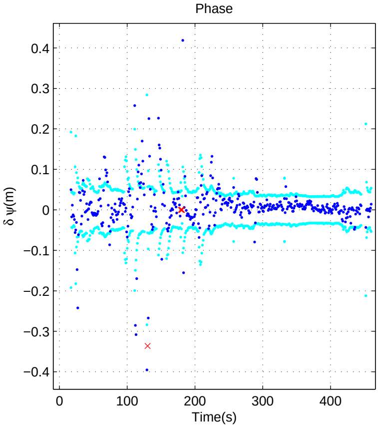
Carrier Phase Differential GPS - INS

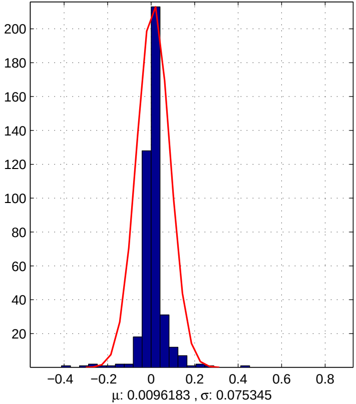
Example EKF Phase residuals for Satellite 3, Stationary rover, baseline: 7 km



Carrier Phase Differential GPS - INS

Example EKF Phase residuals for Satellite 18 driving on I-215:





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CDGPS - Vision - INS

Tightly coupled CDGPS - Vision - INS has several advantages:

- Under some well defined conditions, can contain drifts in:
  - Velocity and gyroscope bias.
  - Certain directions in attitude and accelerometer biases.
- Provides updates even with a single feature (unlike a loosely coupled system).
- Not computationally expensive like SLAM.
- Can be performed in real-time unlike Bundle adjustment.
- Can naturally extend to applications like Mapping, Surveying etc.

"Need to calibrate transformation from Body to Camera frames".

$$\delta \boldsymbol{x}_*^\top = \left[\begin{array}{cccccc|cc} {}^n\delta\boldsymbol{p}_{nb}{}^\top & {}^n\delta\boldsymbol{v}_{nb}{}^\top & {}^n\boldsymbol{\rho}^\top & {}^b\delta\boldsymbol{b}_g^\top & {}^b\delta\boldsymbol{b}_a^\top & & {}^b\delta\boldsymbol{p}_{bc}^\top & {}^b\boldsymbol{\rho}^\top \end{array} \right] \in \mathbb{R}^{21}.$$

CDGPS - Vision - INS

- Feature vector in the Camera frame ${}^{c}\boldsymbol{p}_{cf_j}^{\top} = \begin{bmatrix} x_j & y_j & z_j \end{bmatrix}$.
- Ideal perspective projection model:

$$ {}^{c}\boldsymbol{q}_{cf_j}^{\top} = \begin{bmatrix} u_j & v_j \end{bmatrix} = \frac{1}{z_j} \begin{bmatrix} x_j & y_j \end{bmatrix} \qquad (5.1) $$

- Non-ideal Camera model [2]:

$$ {}^{c}\tilde{\boldsymbol{q}}_{cf_j}^{\top} = \begin{bmatrix} f_x x_j' + c_x & f_y y_j' + c_y \end{bmatrix} + \boldsymbol{n}_c \qquad (5.2) $$

where

$$ \begin{aligned} x_j' &= u_j(1 + k_1 r^2 + k_2 r^4) + 2p_1 u_j v_j + 2p_2(r^2 + 2u_j^2) \\ y_j' &= v_j(1 + k_1 r^2 + k_2 r^4) + 2p_2 u_j v_j + 2p_1(r^2 + 2v_j^2) \\ r &= \|{}^{c}\boldsymbol{q}_{cf_j}\|_2 \end{aligned} $$

Observability Analysis

Key results [14]:

Proposition 1

Assuming that the Camera is fully calibrated (i.e. $\left({}^{b}\mathbf{p}_{bc}, {}^{b}_{c}\mathbf{R}\right)$ are known), then the INS error state $\delta\bar{\mathbf{x}}(0)$ is fully observable with $N_0 \geq 3$ measurements at 3 time instants such that the set of points $\{{}^{n}\mathbf{p}_{nf_0}, \ldots, {}^{n}\mathbf{p}_{nf_{N_0}}, {}^{n}\mathbf{p}_{nc_k}\}$ are not coplanar for all $0 \leq k \leq 2$.

Proposition 2

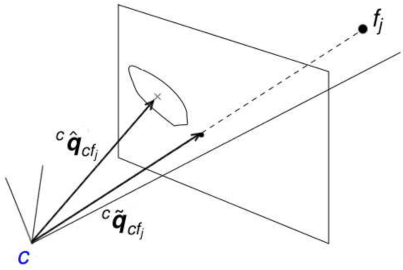
If the rover, initially at rest, is accelerates along a straight line and comes back to rest, aided by both GPS and Vision with $N_0 \geq 3$ features, then the observability gramian has full column rank. Therefore we have full state observability.

CDGPS-Vision-INS

Demo

CDGPS - Vision - INS

Data association:



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Stationary updates - INS

- Reset velocity (${}^{b}\boldsymbol{v}_{nb} = \mathbf{0}$) and rotation (${}^{b}\boldsymbol{\omega}_{nb} = \mathbf{0}$) to zero when system is at rest.

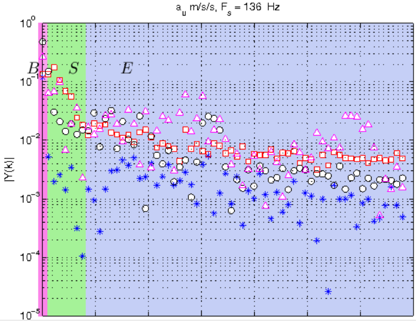
$$\delta\bar{\mathbf{x}}^\top = \begin{bmatrix} {}^{n}\delta\boldsymbol{p}_{nb}{}^\top & {}^{n}\delta\boldsymbol{v}_{nb}{}^\top & {}^{n}\boldsymbol{\rho}_{nb}^\top & {}^{b}\delta\boldsymbol{b}_{g}^\top & {}^{b}\delta\boldsymbol{b}_{a}^\top \end{bmatrix}$$

- Given stationarity, stationary updates or zero updates are preferable.
- Stationary updates corrects errors in:
  - velocity
  - gyroscope biases
  - some linear combination of attitude and accelerometer biases.
- Helps contain errors in position.
  - position
- *Detection of stationarity is a challenge.*
  - False detection introduces errors in sensor bias estimates directly.

Stationary updates - INS

Measurement model:

$$^{b}\tilde{\mathbf{y}}_i = \mathbf{s}(iT_s) + \mathbf{e}(iT_s) + {}^{b}\mathbf{b}(iT_s) + \boldsymbol{\nu}(iT_s) + \mathbf{n}(iT_s)$$



Component	Region in the DFT (f Hz)
$\boldsymbol{B}$	0
$\boldsymbol{E}$	$[8, 85]$
$\boldsymbol{S}$	$(0, 10)$

Legend:

Symbol	:	Maneuver
Blue asterisks	:	Stationary
Red squares	:	Decelerating
Black circles	:	Accelerating
Magenta triangles	:	Constant speed

Stationary updates - INS

- For each sensor k, choose appropriate $m(k) \in \boldsymbol{S}$ [13].
- ${}^k\varphi_{m(k)} \in \mathbb{C}$, ${}^k\varphi_{m(k)} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{P})$.
- Define $f : \mathbb{R}^2 \to [0, +\infty)$ as

$$f({}^k\varphi_{m(k)}) = {}^k\varphi_{m(k)}^\top \boldsymbol{P}^{-1}{}^k\varphi_{m(k)}$$

- Under stationarity, $f({}^k\varphi_{m(k)})$ is an i.i.d. Rayleigh random process with $\chi = 1$.

Stationarity test

Given a chosen harmonic, $m(k) \in \boldsymbol{S}$, for each sensor k, the rover is stationary if

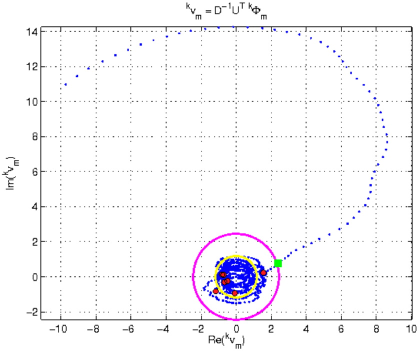
$$\max_{k \in \{1,\ldots,6\}} f({}^k\varphi_{m(k)}) < \lambda^2$$

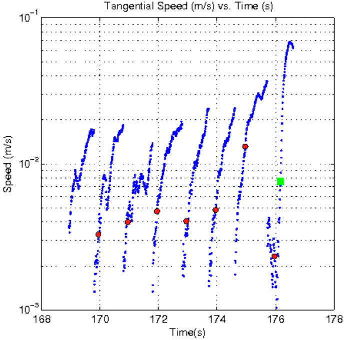
for a chosen threshold $\lambda \in \mathbb{R}^+$.

Stationary updates - INS

- Enables determination of λ using stochastic principles:
  - If $\lambda = 2.4477$, the $p\{\max_{k \in \{1,\ldots,6\}} f({}^{k}\varphi_{m(k)}) < \lambda^2\} = 0.7351$.
  - Conservative choice of $\lambda = 0.05$ resulting in $p_{d/s} = 0.001$ (1 detection every 5 s when $F_s = 130$ Hz).
- Upper bounds on probability of false detection [13].

Stationary updates - INS





Stationary updates - INS

Observability analysis [15]:

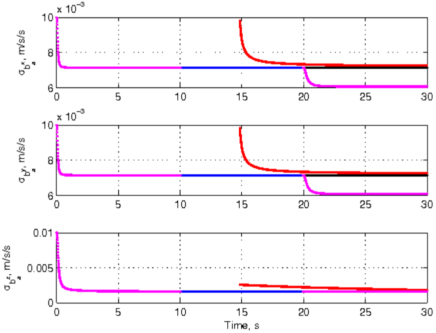
$\delta \boldsymbol{x}$	S	$M - S$	$S - M - S, (\omega = 0)$	$S - M - S$
${}^n\delta \boldsymbol{p}_{nb}$	$\mathbb{R}^3$	$\mathbb{R}^3$	$\mathbb{R}^3$	$\mathbb{R}^3$
${}^n\delta \boldsymbol{v}_{nb}$	$\mathbf{0}$	$\boldsymbol{M}(t_1)\boldsymbol{e}_i$	$\mathbf{0}$	$\mathbf{0}$
${}^n\boldsymbol{\rho}_{nb}$	$\boldsymbol{e}_i$	$\boldsymbol{e}_i$	$\boldsymbol{e}_i$	${}^n\boldsymbol{g}$
${}^b\delta \boldsymbol{b}_g$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
${}^b\delta \boldsymbol{b}_a$	${}^b_n\boldsymbol{R}[{}^n\boldsymbol{g}\times]\boldsymbol{e}_i$	${}^b_n\boldsymbol{R}(t_1)[{}^n\boldsymbol{g}\times]\boldsymbol{e}_i$	${}^b_n\boldsymbol{R}[{}^n\boldsymbol{g}\times]\boldsymbol{e}_i$	$\mathbf{0}$

Fixed point optimal smoother:

$$\delta \hat{\boldsymbol{x}}_0 = E\{\delta \hat{x}_0 | \delta \boldsymbol{y}_1 \ldots \delta \boldsymbol{y}_M\}.$$

Stationary updates - INS

$\delta\boldsymbol{x}$	S	$M-S$	$S-M-S,(\omega=0)$	$S-M-S$
${}^{b}\delta\boldsymbol{b}_a$	${}^{b}_{n}\boldsymbol{R}[{}^{n}\boldsymbol{g}\times]\boldsymbol{e}_i$	${}^{b}_{n}\boldsymbol{R}(t_1)[{}^{n}\boldsymbol{g}\times]\boldsymbol{e}_i$	${}^{b}_{n}\boldsymbol{R}[{}^{n}\boldsymbol{g}\times]\boldsymbol{e}_i$	$\boldsymbol{0}$



Legend:

Color	:	Maneuver
Blue	:	S
Red	:	$M-S$
Black	:	$S-M-S(\omega=0)$
Magenta	:	$S-M-S(\omega\neq 0)$

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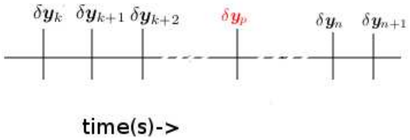
Near Real Time estimation



- Unlikely residuals: $\delta \boldsymbol{y}_p^\top \boldsymbol{S}_p^{-1} \delta \boldsymbol{y}_p > \lambda$.
- Validation from measurements in future time $\tilde{\boldsymbol{y}}_n, \tilde{\boldsymbol{y}}_{n+1}, \dots$
- Updating $\delta \bar{\mathbf{x}}_n$ with $\delta \boldsymbol{y}_p$ violates “white-noise” assumption.

A possible solution:

- Append state vector: $\delta \boldsymbol{\alpha}^\top = \begin{bmatrix} \delta \bar{\mathbf{x}}_p^\top & \delta \bar{\mathbf{x}}_n^\top \end{bmatrix}$.
- Append the covariance: $\boldsymbol{P}_\alpha = \begin{bmatrix} \boldsymbol{P}_p & E\{\delta \bar{\mathbf{x}}_p \bar{\mathbf{x}}_n^\top\} \\ E\{\bar{\mathbf{x}}_n \delta \bar{\mathbf{x}}_p^\top\} & \boldsymbol{P}_n \end{bmatrix}$.
- Update: $\delta \boldsymbol{y}_p = \begin{bmatrix} \boldsymbol{h}_p & \mathbf{0} \end{bmatrix} \delta \boldsymbol{\alpha}$.

Thanks for listening!

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