“Kalman Filter Implementation with Improved Numerical Properties”

Accepted for publication in IEEE Transactions on Automatic Control

Dr Mohinder Grewal and James E. Kain

NUMBER FP-09-062.1
Dr Mohinder Grewal

Professor of Electrical Engineering  California State University, Fullerton
where he received the 2009 Outstanding Professor award.
Define the problem

Ideas, Models, Measurements, Performance Goals

Possible solutions

Simulate, implement & test with high-level code

Production Implementation

Hardware and low-level code 100% reliability
Closing the Design>Implementation Gap

MATRXx – System Design and Control Development Software

An Example
Closing the Design-Implementation Gap

Algorithmic Acceleration Through Automated Generation of FPGA Coprocessors

C-to-FPGA design methods allow rapid creation of hardware-accelerated embedded systems.
Common Kernels as Building Blocks for all Algorithms

**View From Berkeley**

**Applications**
1. What are the applications?
2. What are common kernels of the applications?

**Programming Models**
5. How to describe applications and kernels?
6. How to program the hardware?

**Evaluation:**
7. How to measure success?

**Hardware**
3. What are the hardware building blocks?
4. How to connect them?

_Tension between Embedded & Server Computing_

**Figure 1.** A view from Berkeley: seven critical questions for 21st Century parallel computing. (This figure is inspired by a view of the Golden Gate Bridge from Berkeley.)
"The dwarfs present a method for capturing the common requirements of classes of applications while being reasonably divorced from individual implementations."

1. Dense Linear Algebra
2. Sparse Linear Algebra
3. Spectral methods
4. N-Body Methods
5. Structured grids
6. Unstructured grids
7. Monte Carlo Methods

1970's Promise of Modern System Theory

Ideas, Models, Measurements, Performance Goals

- Stochastic Models
- Measurement Models
- Error Statistics

THE Optimal Solution

Accelerate the Engineering Process!
Why Isn’t the Kalman Filter used for Signal Processing?
Possible Reasons...

- After 30 years of implementations, robustness concerns remain
  - Square root filtering
  - Fictitious noise injection
  - Fading memory filters
  - Initialization issues
  - Residual tracking/adaptation
  - “Tuning”
- No straightforward path to fixed point
Propagate standard deviation (\( \sigma \)) and correlation coefficients (\( \rho \)) rather than covariance matrix or square root factors

- Square root-like behavior
- Natural scaling options
- Physical interpretation

Sigma and Rho trends provide information on current and projected robustness status

- Rate of convergence
- Limit of convergence
- Correlation properties
### Continuous System Propagation

#### Auxiliary Variables

<table>
<thead>
<tr>
<th>Auxiliary variables</th>
<th>$\sigma'<em>i = \frac{\sigma_i}{\sigma</em>{\text{Max}_i}}$</th>
<th>$F'<em>{ij} = \frac{F</em>{ik}\sigma_{\text{Max}<em>k}}{\sigma</em>{\text{Max}_i}}$</th>
<th>$Q'<em>{ij} = \frac{Q</em>{ij}}{\sigma_{\text{Max}<em>i} \sigma</em>{\text{Max}_j}}$</th>
</tr>
</thead>
</table>

#### Standard Deviation Propagation

<table>
<thead>
<tr>
<th>Standard Deviation Propagation</th>
<th>$\frac{\dot{\sigma}'<em>i}{\sigma'<em>i} = M'</em>{ii} + \frac{Q'</em>{ii}}{2\sigma'^2_{ii}}$</th>
<th>$M'<em>{ij} = \sum</em>{k=1}^{n} F'_{ik}\sigma'<em>k \rho</em>{kj} \sigma'_i$</th>
</tr>
</thead>
</table>

#### Correlation Coefficient Propagation

<table>
<thead>
<tr>
<th>Correlation Coefficient Propagation</th>
<th>$\dot{\rho}_{ij} = - \left( \frac{\dot{\sigma}'<em>i}{\sigma'<em>i} + \frac{\dot{\sigma}'<em>j}{\sigma'<em>j} \right) \rho</em>{ij} + M'</em>{ij} + M'</em>{ji} + \frac{Q'</em>{ij}}{\sigma'_i \sigma'_j}$</th>
</tr>
</thead>
</table>

#### State Propagation

<table>
<thead>
<tr>
<th>State Propagation</th>
<th>$\dot{x}'<em>i = \frac{f(x)}{\sigma</em>{\text{Max}_i} \sigma'_i} - x'_i \frac{\sigma'_i}{\sigma'_i}$; $x_k = x'<em>k \sigma</em>{\text{MAX}_k} \sigma'_k$</th>
</tr>
</thead>
</table>

**Discrete System Propagation**

<table>
<thead>
<tr>
<th>Normalized Dynamics</th>
<th>$x_i' = \frac{\lambda x_i}{\sigma_{\text{Max}} \sigma_i'}$; $\Phi_{ij}' = \frac{\Phi_{ij} \sigma_{\text{Max}}}{\sigma_j' \sigma_i'}$; $Q_{kij}' = \frac{Q_{kij}}{\sigma_{\text{Max}} \sigma_{\text{Max}} \sigma_i' \sigma_j'}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation Propagation</td>
<td>$\left( \frac{\sigma_i'}{\sigma_i'^{-}} \right) = \sqrt{\Phi' \Gamma \Phi'^T + Q_k'}<em>{ii}$; $\Gamma</em>{ij} = \rho_{ij}^+$</td>
</tr>
<tr>
<td>Correlation Coefficient Propagation</td>
<td>$\rho_{ij}^{-} = \left( \frac{\sigma_i'^+}{\sigma_i'^{-}} \right) \left( \frac{\sigma_j'^+}{\sigma_j'^{-}} \right) \left[ \Phi' \Gamma \Phi'^T + Q_k' \right]_{ij}$</td>
</tr>
<tr>
<td>State Propagation</td>
<td>$x_{i}^{-}(k+1) = \left( \frac{\sigma_i'^+}{\sigma_i'^{-}} \right) \left[ \Phi' x'(k) \right]_i$</td>
</tr>
</tbody>
</table>
# Measurement Update

<table>
<thead>
<tr>
<th>Auxiliary Parameters</th>
<th>[ D_i = \sum_{k=1}^{n} H_k' \sigma_k' \rho_{ki}^- ; H_i' = H_i \sigma_{Max_i} ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual Variance</td>
<td>[ \Omega^2 = \sum_{k=1}^{n} D_k \sigma_k' H_k' + R ]</td>
</tr>
<tr>
<td>Standard Deviation Update</td>
<td>[ \frac{\sigma_i'^+}{\sigma_i'^-} = \sqrt{1 - \left[ \frac{D_i}{\Omega} \right]^2} ]</td>
</tr>
<tr>
<td>Correlation Coefficient Update</td>
<td>[ \rho_{ij}^+ = \left[ \frac{\sigma_i'^-}{\sigma_i'^+} \right] \left[ \frac{\sigma_j'^-}{\sigma_j'^+} \right] \left[ \rho_{ij}^- - \frac{D_i D_j}{\Omega^2} \right] ]</td>
</tr>
<tr>
<td>State Update</td>
<td>[ x_i'^+ = \left[ \frac{\sigma_i'^-}{\sigma_i'^+} \right] \left{ x_i'^- + \frac{D_i}{\Omega} \left[ \frac{\lambda z_m - H_k' \sigma_k' x_k'}{\Omega} \right] \right} ]</td>
</tr>
</tbody>
</table>
Filter Adaptation

Measurement Noise Adaptation
Limit State improvement at a measurement

\[
\frac{\sigma_i'^+}{\sigma_i'^-} = \sqrt{1 - \left[ \frac{D_i}{\Omega} \right]^2}
\]

Test

Process Noise Adaptation (surgical control)
Limit correlation coefficients and lower sigma limit

Computed

\[
\rho_{ij}' = \frac{\rho_{ij}}{1 + \gamma} \quad \sigma_i'^2 = \sigma_i^2(1 + \gamma)
\]

Adapted
Uncertain frequency model

\[ \frac{1}{s^2 + 2 \xi_i \omega_i s + \omega_i^2} \]

Carrier frequency model

\[ \frac{1}{s^2 + 2 \xi_c (\omega_c + x_1) s + (\omega_c + x_1)^2} \]
BPSK Encoding onto Carrier

Class 1

Class 2

PSK Data encoder

Phase transitions

Composite signal with encoded data
Carrier Model Design

(a) Spectral response

(b) Autocorrelation
Significant state nonlinearity!

\[
\begin{bmatrix}
\dot{x}_0 \\
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\end{bmatrix} =
\begin{bmatrix}
-2\xi_i \omega_i & -\omega_i^2 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & -2\xi_c (\omega_c + x_1) & - (\omega_c + x_1)^2 \\
0 & 0 & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
x_0 \\
x_1 \\
x_2 \\
x_3 \\
\end{bmatrix}
+
\begin{bmatrix}
w_i \\
0 \\
w_c \\
0 \\
\end{bmatrix}
\]
Fortunately!

Transition Matrix and Discrete Noise matrix can be solved in closed form with high accuracy

Assumptions:

• Information 2\textsuperscript{nd} order model not influenced by carrier model

• Information bandwidth << carrier frequency
# Model Parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier frequency</td>
<td>1.5 GHz</td>
</tr>
<tr>
<td>Carrier damping</td>
<td>$10^{-8}$</td>
</tr>
<tr>
<td>Information bandwidth</td>
<td>1.0 Hz</td>
</tr>
<tr>
<td>Information model damping</td>
<td>0.50</td>
</tr>
<tr>
<td>Sample rate</td>
<td>74MHz (1/20.5 carrier)</td>
</tr>
<tr>
<td>Encoding rate</td>
<td>7.5MHz (1/200 carrier)</td>
</tr>
<tr>
<td>Noise</td>
<td>10% carrier amplitude</td>
</tr>
<tr>
<td>Initial carrier frequency error</td>
<td>50 PPM</td>
</tr>
<tr>
<td>Carrier frequency error sigma</td>
<td>100 PPM</td>
</tr>
<tr>
<td>Information gain limit</td>
<td>50% / measurement</td>
</tr>
<tr>
<td>Sigma lower limit</td>
<td>10% initial value</td>
</tr>
<tr>
<td>Correlation limit</td>
<td>+/- 0.95</td>
</tr>
</tbody>
</table>
Within N-sigma threshold

Change model to reflect new class
- states
- correlations

Within N-sigma threshold

Process this measurement

Normalized residual

Two-class Hypothesis Test provides maximum Likelihood estimate of correct class

Data bit change
Representative Results
BPSK Data Transitions
Transition to Fixed Point

Use floating point implementation to define mantissa
- Replace all multiply & divide operations with a special procedure that tracks the maximum range of all internal variables
- Adjust scale factors so that range is +/- 2.0

Test Fixed point model
Replace all multiply and divide operations by
N = number of fraction bits

Multiply: \[ C2^N \] = ROUND \left( \begin{bmatrix} A2^N \\ B2^N \end{bmatrix} 2^{-N} \right)

Divide: \[ C2^N \] = ROUND \left( \frac{A2^N}{B2^N} \right)

Integer SigmaRho Filter
12 fixed point bits 10 fraction bits
Integer SigmaRho Filter
10 fixed point bits 8 fraction bits
100MHz sampling to decode data from carrier

- Direct carrier sampling
- De-spreading still required for initial CDMA lock & noise removal
- Replace frequency and code tracking with model-based KF
- Hypothesis testing for BPSK or QPSK or other encoding
- OFDM using multiple carrier models

SigmaRho fixed point filter makes VDLL practical

Leap forward SDR GPS + 3G/4G Comm

- Directly sampled carriers with software
- VDLL GPS
- Deeply embedded MEMS
- OFDM comm
Why IMU/GPS?

- **Make GPS Better**
  - ✓ Feedforward to Tighten Track loops
    - ◆ Densensitization to accel
    - ◆ Low C/N0 operation
    - ◆ Multipath rejection
  - ✓ Smooth noisy position
  - ✓ Operate through GPS outages

- **6DOF Vehicle Characterization**
- **Measure attitude**
  - ✓ Antenna pointing
  - ✓ Remote Sensing
Attitude Enables Antenna Pointing

Teledesic Enables Connected Car PC
Attitude Enables Georegistration
Georegistration Enables Change Detection

Prescription Application

June 15, 2002  June 30, 2002
La Cumbre
Argentina
International Operations
La Cumbre
Argentina
Large Format Camera Pan-sharpened

Flown October 5, 10:00AM

GeoVantage

Flown October 5, 10:30AM

GeoVantage & Large Format simultaneous data collection

Flown October 5, 2008

Resolution Six inch
Geoscanner Digital Camera System

- Digital Cameras
- IMU
- GPS and electronics
Single Frame GeoRegistration
WITHOUT Ground Control

Critical Factors:
- Camera/aircraft position
- Camera Attitude*
- Camera model
- Camera-IMU alignment
- DEM

* 0.1 deg roll err => 6.4m error @ 12000 ft alt
Extensive IMU Integration Experience

Honeywell HG1700
Northrup Grumman LN200

AIS WCMD
AIS AIMU

Enpoint MEMS

GeoPod
Flight Operations Improve Attitude Estimation

Technology and demand pushing imagery resolution to “inches” with equivalent georegistration accuracy

Can improve georegistration by:

- Lower flight altitude
- Short flight lines
- Steep turns
Representative Aircraft Dynamics

8 mi
Kalman Filter Attitude Estimation

Roll attitude

Backward Filter roll error
Forward Filter roll error

Composite roll error

Roll error (mrad)

Elapsed time (sec)
Seam Correction: Internal Tiepoints

Imagery with Seams is Unacceptable to clients

Solution:

- Place candidate tiepoints on single frames
- Locate match points on overlapping frames
- Adjust single frames for registration
- Use tiepoints for tonal balance
Candidate Tiepoint Selection

- Accepted tiepoint
- Rejected (low contrast)
- Rejected (bad correlation peak)
Candidate Tiepoints
Massive Use of Tiepoints

Complete automation:
40,000 matched tiepoints selected from over 400,000 candidate tiepoints
# IMU Specifications

<table>
<thead>
<tr>
<th>Vendor and Model</th>
<th>Max Gyro Rate (deg/s)</th>
<th>Gyro In-run Bias (deg/hr)</th>
<th>Gyro SFE (ppm)</th>
<th>Angle Random Walk (deg/rt-hr)</th>
<th>Max Accel (g)</th>
<th>Accel In-run Bias (mg)</th>
<th>Accel SFE (ppm)</th>
<th>Velocity Random Walk (m/s/rt-hr)</th>
<th>Quantity Price ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atlantic Inertial AIMU*</td>
<td>1400</td>
<td>2</td>
<td>350</td>
<td>0.05</td>
<td>40</td>
<td>0.3</td>
<td>700</td>
<td>0.03</td>
<td>11000</td>
</tr>
<tr>
<td>Honeywell HG1700*</td>
<td>1074</td>
<td>1</td>
<td>150</td>
<td>0.13</td>
<td>37</td>
<td>1.0</td>
<td>300</td>
<td>0.02</td>
<td>18500</td>
</tr>
<tr>
<td>Northrup Grumman LN200*</td>
<td>1000</td>
<td>1</td>
<td>100</td>
<td>0.07</td>
<td>40</td>
<td>0.3</td>
<td>300</td>
<td>0.08</td>
<td>27000</td>
</tr>
<tr>
<td>Analog Devices ADIS16365</td>
<td>300</td>
<td>25</td>
<td>1000</td>
<td>1.90</td>
<td>17</td>
<td>0.2</td>
<td>3000</td>
<td>0.35</td>
<td>300</td>
</tr>
</tbody>
</table>

* ITAR control required
Access to Commercial MEMS Navigation
MEMS IMU Hardware Features

- ADI ADIS1635x
  - 3-axes acceleration measurement
  - 3-axes attitude rate measurement
- ADI Blackfin BF533 or BF538 for application support
- 3-axis magnetometer
- SDRAM – up to 32MB
- Flash – up to 2MB
- Protection: over voltage, ESD, polarity
Traditional Tiepoint Mechanization
Three sequential steps

- **Navigation**
  - IMU model
  - IMU data
  - GPS Position
  - Correct IMU errors
  - Navigation Kalman filter

- **Projection**
  - Single frame projection
  - Tiepoint selection
  - Tiepoint match
  - Single frame reprojection
  - Bundle adjustment

- **Mosaic**
Imagery Aided Navigation

Coupled Kalman filter

Predict position

Predict tiepoints

GPS Position

Position & attitude

Tiepoint error

Correct IMU errors

Image data

Camera model

Single frame projection

Tiepoint selection

Tiepoint match

Mosaic
Navigation Aiding from Mosaicked Imagery

\[
p = \begin{bmatrix} fl \\ \varepsilon Y_{\text{pix}} \\ \varepsilon Z_{\text{pix}} \end{bmatrix} / \sqrt{f^2l^2 + (\varepsilon Y_{\text{pix}})^2 + (\varepsilon Z_{\text{pix}})^2}
\]

\[R = \frac{h_a - h_g}{\left[C_b^e(t_1)p_1\right] \cdot \hat{g}}\]

\[
\begin{bmatrix}
\dot{Y}_{\text{pix}} \\
\dot{Z}_{\text{pix}}
\end{bmatrix} = \begin{bmatrix}
\begin{array}{c|c|c}
\begin{array}{c}
p_2 \\
p_1 \\
p_3 \\
p_1
\end{array} & \begin{array}{c}
1 \\
p_1 \\
0 \\
p_1
\end{array} & \begin{array}{c}
0 \\
0 \\
1 \\
p_1
\end{array}
\end{array}
\end{bmatrix}
\begin{bmatrix}
\frac{\dot{r}_a}{R} + \frac{(r_a - r_{\text{TP}})}{R} \cdot \frac{\dot{r}_a}{R} p + \omega_{b/e}^b \times p
\end{bmatrix}
\]

Propagate tiepoint to next observance
SigmaRho Efficiency for Large State Vectors
GeoVantage GeoPod Design

Goal: 10000 pixel crossrange, non-ITAR

- Splayed Canon T1i Cameras
  - 9700 pixels crossrange
  - 50 deg FOV

- Existing electronics & IMU

- Customization Board

- GPS RF

- Data & power

- Existing STC aircraft interface

- Environmentally sealed & vibration isolated
Aircraft Flight Computer

Web Information Flow

- Data Request
- Data status
- User Help
- Feedback

Internet

- Mission status
- Hardware status
- QC metrics

Aviation Operations

- Mission Plans
- Processing commands

Imagery Products

- Data Request
- Data status
- User Help
- Feedback

Land Use Managers
GeoVantage Business Goals

- Scale to 1000 RS systems
- Heavy International Focus
- Low Price/area (below satellite)
- Fast Response Times (<one day)
- Web Order-to-Delivery with web status
- Standardized RS Products
- Automate Everything